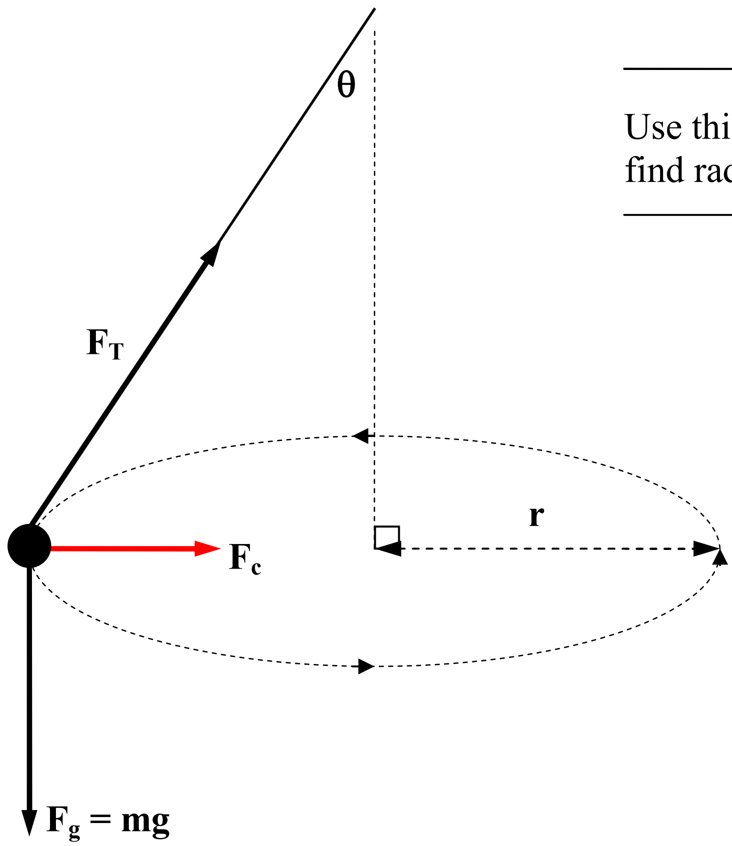
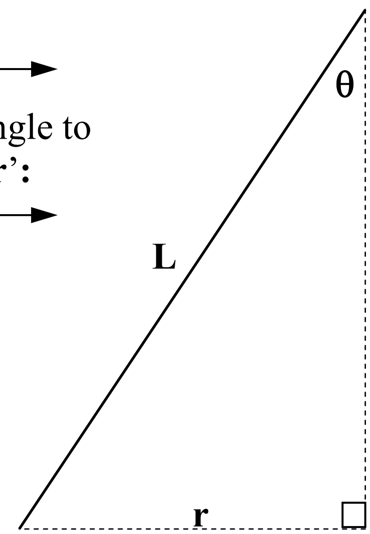
Analysis of Force Vectors in Two Dimensions.

Now we examine circular motion caused by forces that aren't acting on the same plane as the circle.

A: The Conical Pendulum



Use this triangle to find radius '**r**':



$\mathbf{\sin\theta = r/L}$

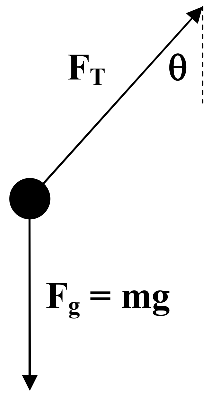
$\mathbf{r = L\sin\theta}$

where **r** is the radius of the circular path and **L** is the pendulum length

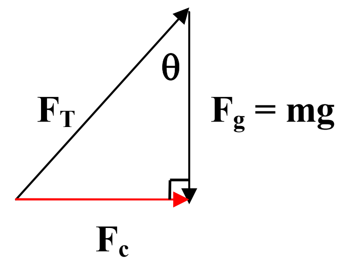
Examine the above diagram and make note of the vectors drawn.

In this situation, there are only two forces acting on the pendulum bob: the tension force $\mathbf{F_T}$ and the weight of the bob $\mathbf{F_g}$. The centripetal force $\mathbf{F_c}$ is actually a *net force*, the resultant sum of the two previous forces mentioned, as shown on the next page:

The free-body diagram:



→ add these vectors and draw the resultant $\mathbf{F_c}$ →



A right triangle is the result. Use trigonometry, along with

$$\mathbf{F_c = m \frac{4\pi^2 r}{T^2}} \qquad \text{and} \qquad \mathbf{F_c = m \frac{v^2}{r}}$$

to find tension $\mathbf{F_T}$, speed $\mathbf{v}$, period $\mathbf{T}$, etc.

Example #8: A 140 g ball is fastened to one end of a 0.24 m string, and the other end is whirled in a horizontal conical pendulum. Find:

a) the speed of the ball in its circular path;

b) the tension in the string that makes an angle of 30º to the vertical.

(see Circular Motion Ex 8 for answer)

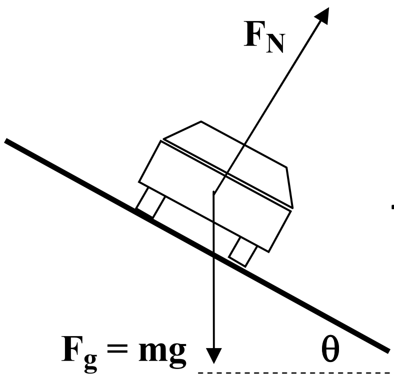
B: Banked Curves

The purpose of banking a roadway, especially on an oval racetrack where cars are travelling at high speeds, is so that the bank itself can provide all the turning force. In this way, no friction is needed and accidents are less likely to occur.

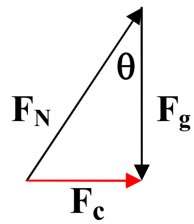
In other words, for a particular speed, the car can steer itself around the corner and never miss a turn because of ice. This is the *null speed*, the speed at which the curve is said to have "ideal banking" or "proper" banking.

Note that the equation for null speed can be derived. At this speed, there are only two forces (the weight and the normal force) which add together to supply the centripetal force ($\mathbf{F_c}$):

Free-body diagram of car on a curved bank, coming towards you (out-of-page) and turning to the right:



→vector-add to find the resultant:



To the right, the sum of the two forces produces a right triangle with resultant $\mathbf{F_c}$, just as in the conical pendulum. From this diagram, we see that:

$\tan\theta = \frac{F_c}{mg}$ where $F_c = m\frac{v^2}{r}$ and '**r**' is the radius of the track's curve.

→ substitute and cancel to obtain $\tan\theta = \frac{v^2}{rg}$

Note that this derivation is based on the null speed only (i.e. no friction).

Example #9: A curve of 30 m radius is banked so that a car may make a turn at a speed of 13 m/s without depending on friction at all. What is the slope of the curve?

(see Circular Motion Ex 9 for answer)

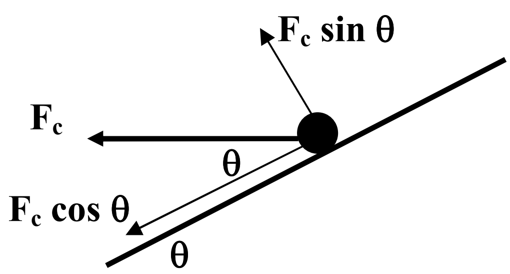
What would occur if you were driving a car around an "ideally" banked corner at a speed:

a) below the null speed?
   - if the car is travelling too **slowly**, the normal force of the track will not be great enough to hold the car in a circular path, so the car will start to slide down the bank;
   - therefore, friction must act *up*-bank to keep the vehicle in circular motion.

b) greater than the null speed?
   - if the car is travelling too **quickly**, its momentum would cause the car to start sliding up the bank;
   - therefore, a friction force must act *down*-bank to prevent this motion and keep the vehicle moving in a circular path.

In either case, there are too many vectors to create a triangle, and is therefore beyond the scope of this course. However, if you're finding this material much too simple and need a challenge, read on!

Examine force components both parallel and perpendicular to the surface of the bank.



→$\mathbf{F_c \cos \theta}$ is the centripetal force parallel to the road bank

→$\mathbf{F_c \sin \theta}$ is the centripetal force $\perp$ to the road bank

Examining the perpendicular forces,

- $\mathbf{F_c \sin \theta}$ is the unbalanced force which supplies the perpendicular component of $\mathbf{a_c}$.
- The normal force ($\mathbf{F_N}$) acts positively to supply this net force, while the perpendicular component of the car's weight ($\mathbf{F_g \cos \theta}$) acts against it, so that:

$$\mathbf{F_c \sin \theta = F_N - F_g \cos \theta} \qquad \textbf{(1)}$$

Examining the parallel forces,

- we see that inertia would carry the mass up-slope as it rounds the bank; an unbalanced force, $\mathbf{F_c \cos \theta}$ exists which keeps this from happening.
- The parallel component of the car's weight ($\mathbf{F_g \sin \theta}$) will act *positively* down-slope to supply this net force. As well,
  - if the car is travelling too slowly, friction will act up-slope, *negatively*, so that:

$$\mathbf{F_c \cos \theta = F_g \sin \theta - F_f} \qquad \mathbf{(2)}$$

  - if the car is travelling too quickly, friction will act down-slope, *positively*, so that:

$$\mathbf{F_c \cos \theta = F_g \sin \theta + F_f} \qquad \mathbf{(3)}$$

Use either equation (1) and (2) or (3) by substituting in $\mathbf{F_c = m \frac{4\pi^2 r}{T^2}}$ and $\mathbf{F_c = m \frac{v^2}{r}}$, as well as $\mathbf{F_f = \mu F_N}$ to solve for unknown values.

Remember, this is a university-style problem, so if you're having difficulty with it, give it a pass, since it is unlikely to show up on the final exam.

Example #10: The radius of a velodrome curve is 40 m and the banked angle is 15º. If μ=0.20, what is the maximum speed at which a cyclist can take this curve without slipping?

(see Circular Motion Ex 10 for answer)